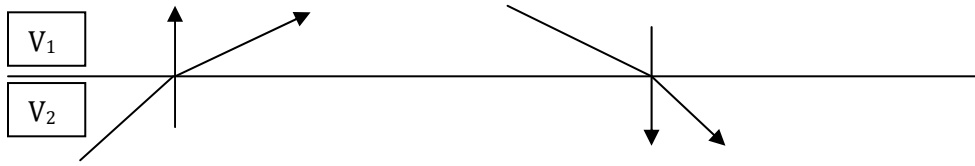


Seismology and Global Waves Chap. 4 HW Answers

1. A swinging door is embedded in a N-S oriented wall. What force direction would be required to make the door swing on its hinge?
2. A ray is travelling in a rock with a 3.00 km/s seismic velocity. The ray encounters an interface at a 45.0° angle with a rock velocity of 4.00 km/s on the other side. At what angle with respect to the normal to the interface does the ray leave the interface?

Name the incidence angle θ_1 and solve for the transmitted ray angle θ_2



Snell's Law: $\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2} \triangleright \sin(\theta_2) = \frac{v_2}{v_1} \sin(\theta_1) \triangleright \theta_2 = \sin^{-1}\left(\frac{v_2}{v_1} \sin(\theta_1)\right)$

Substitute in values: $\theta_2 = \sin^{-1}\left(\frac{4}{3} \sin(45^\circ)\right) = 70.5^\circ$

Q: does the ray bend towards or away from the outgoing side of the surface normal.

Q: what is the y value when the absolute value of the argument $x = \left\| \frac{v_2}{v_1} \sin(\theta_1) \right\|$ to the inverse sine function $y = \sin^{-1}(x)$ is > 1 .

Q: Reverse the direction that the ray travels and calculate the outgoing ray angle with respect to the surface normal.

3. A ray approaches an interface at an angle of 10° from the interface. What is the angle with respect to the surface normal? What angles are used to evaluate Snell's Law formula: the surface normal or the acute or obtuse angle with respect to the interface?
4. If a ray arrives at the core-mantle boundary at an angle of 25°, at what angle does the ray enter the core?

Think. The rays are propagating in a sphere (r, θ) , not in a Cartesian (x/y) geometry. So, the spherical ray tracing equation (4.8 page 30) should be used. Define the incoming angle as $\theta_1=25^\circ$ and the outgoing angle that propagates the ray in the core as θ_2 . Note that we are not propagating the ray across a spherical shell layer (Fig. 4.13), hence r_1 and r_2 are the same value which is the radial depth to the core mantle boundary (2900 km).

$$\frac{r_1 \sin(\theta_1)}{v_1} = \frac{r_2 \sin(\theta_2)}{v_2}, \text{cancel out } r_1 \text{ and } r_2, \triangleright \theta_2 = \sin^{-1}\left(\frac{v_2}{v_1} \sin(\theta_1)\right)$$

Substitute in the velocities on either side of the boundary (important to get correct order) and the angle of incidence.

$$\theta_2 = \sin^{-1}\left(\frac{7}{13} \sin(25^\circ)\right) = 13.2^\circ \text{ is the angle that the ray enters the outer liquid iron core.}$$

- 5. A ray hits a interface where the velocity increases from 2 km/s to 4 km/s and is refracted along the interface. At what angle does the ray approach the surface?**

If the layer is flat (parallel with respect to the surface), then the angle of incidence of this new type of ray – a refracted ray that travels parallel to the interface – is equal to the angle that the refracted ray departs the interface to propagate upwards. So, let us solve for the angle of incidence for the ray to be refracted horizontal. Note that horizontal propagation of the refracted ray means $\theta_2 = 90^\circ$. Therefore, using Snell's law with $\theta_2 = 90^\circ$ permits θ_1 to be calculated as:

$$\theta_1 = \sin^{-1}\left(\frac{v_1}{v_2} \sin(\theta_2)\right) = \sin^{-1}\left(\frac{2}{4} \sin(90^\circ)\right) = \sin^{-1}\left(\frac{2}{4} * 1\right) = 30^\circ.$$

- 6. What is the quickest time it takes seismic energy to travel the following epicentral angles?**

- (a) $50^\circ \gg 9$ minutes
- (b) $90^\circ \gg 12$ min.
- (c) $98^\circ \gg 13.5$ min.
- (d) $142^\circ \gg 19.5$ min.
- (e) $180^\circ \gg 20$ min.
- (f) $183^\circ \gg 360^\circ - 183^\circ = 177^\circ$

- 7. What are the least and greatest epicentral angles at which a P' ray can arrive from its source?**

Least time $\Delta=143^\circ$

Greatest time $\Delta=180^\circ$

8. A layer (rock) has a compressional wave velocity $V_p=2.5$ km/s and a shear wave velocity of $V_s=1.5$ km/s. If a P-wave ray traveling upwards hits the surface at an incidence angle of 30° with respect to the surface normal, at what angle does the S-ray reflect ?

Important detail for this question is that the incident ray is a P-wave and the reflected ray is a S-wave. This means that in Snell's Law we must place the different velocities in the correct place. Assuming $\theta_1 = 30^\circ$ is the incidence angle, the correct form of Snell's Law is to associate θ_1 with the incident P-wave and to associate θ_2 with the reflected S-wave.

$$\frac{\sin(\theta_1)}{v_p} = \frac{\sin(\theta_2)}{v_s} \triangleright \theta_2 = \sin^{-1}\left(\frac{v_s}{v_p} \sin(\theta_1)\right) \triangleright \theta_2 = \sin^{-1}\left(\frac{1.5}{2.5} \sin(30^\circ)\right) = 17.4^\circ$$

So, the S-wave is reflected at 17.4° with respect to the surface normal.

9. A P-ray reflects from an interface as both P- and S-rays (waves). Compare the angle of reflection of the P-ray with the S-ray.

- (i) The same. Untrue, except for one special angle. Look at Snell's Law and note that if V_p and V_s are different values in the denominator, then the equality can never be made IF the angles are the same. Note that P and S wave velocity are never the same.
- (ii) Always bigger. Untrue. Typical a layer S-wave velocity is only 55% of the layer's P-wave velocity, i.e., $V_s=0.55*V_p$. Substituting the relation between the V_p and V_s in a layer into Snell's Law gives:

$$\frac{\sin(\theta_1)}{v_p} = \frac{\sin(\theta_2)}{v_s} \triangleright \frac{\sin(\theta_1)}{v_p} = \frac{\sin(\theta_2)}{0.55*v_p}$$

To satisfy the equal sign between the right and left hand terms, the inequality - $\sin(\theta_2) < \sin(\theta_1)$ - must be satisfied. Because the sine function is a monotonic function (always increasing from 0° to 90°), then to make the inequality correct, $\theta_2 < \theta_1$. Remember that θ_1 is the incident P-wave and θ_2 is the reflected S-wave and hence the reflected S-wave angle is ALWAYS smaller than incident P-wave angle.

- (iii) Always smaller. Correct answer. See (ii) for complete answer.
- (iv) Sometimes bigger, sometimes smaller. An awful answer as the world would be pure chaos if this was true (random world).

10. An underground explosion should produce all P-waves and NO S-waves (Chap. 5). Yet, S-waves from underground explosions are observed on seismograms. So, how could P-waves energy made by the explosion be converted to S-waves ?

Simple. At any interface where velocity changes and at the earth's surface, P-waves are converted to S-waves. Thus, as the P-waves from an explosion propagate towards the seismometer recording then, they are converted to a small fraction of S-wave energy.

11. A granite has a density at least 2.5 times greater than that of water, but sounds travels faster through it.

(a) Why should a lower density make for a lower velocity ?

Let us start at the beginning.

The **velocity of waves** is equal to:
$$V = \sqrt{\frac{\text{restoring elastic force (stress)}}{\text{mass of parcel}}}$$

The **strength of rocks (modulus)** is defined as: modulus = applied-stress/strain

The **bulk modulus** is defined by the Greek letter kappa as:

$$\kappa = \frac{P}{\left(\frac{dv}{v}\right)}: P \text{ is the pressure applied to the sphere, } v \text{ is velocity, } dv \text{ is velocity change.}$$

This number represents the strength of the rock to when a pressure (Force/area) is applied to the rock parcel.

The **shear (or rigidity) modulus** is define by Greek letter mu as:

$$\mu = \frac{F}{A * d\theta}: F \text{ is applied shear force, } A \text{ is area, } d\theta \text{ shear angle.}$$

This number represents the strength of the rock when a shear stress is applied to the rock parcel.

A compressional (P) wave makes both compressional and shear stress and hence the formula for P-wave velocity contains both the bulk and shear modulus in it.

$$V_p = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$

The shear (S) wave makes NO compressional stress, only shear stress, and hence the formula for the S-wave velocity does NOT contain the bulk modulus in it.

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

So, given that the density term is in the denominator of the V_p and V_s expressions, then increasing the density, while keeping the bulk and shear modulus constant, will cause the velocities to go DOWN!

(b) In what additional way must the properties of granite vary from those of seawater?

Inspection of the velocity equations above show that for the velocity of granite to get greater than water, even though it has greater density, requires that the increases in the granite's bulk and shear modulus to be proportionally larger with respect to the increase in density. Specifically, in the P-wave velocity equation the numerator (top) term must increase more than denominator (bottom) increase in density.

$$\frac{A * (\kappa + \frac{4}{3} \mu)}{B * \rho} \text{ increases } \triangleright \frac{A}{B} > 1$$

12. Consider that the earth has a constant velocity everywhere (uniform). Calculate the travel time ratio for an epicentral angle of 90° and 60°.

When the seismic velocity of a body (e.g., earth) is constant, the shortest time path for a seismic ray is a straight line (Fig. 4.8 pg. 28). In contrast, if the earth's velocity is variable (changes in 3-D), then the shortest time path between an earthquake and seismometer will NOT be a straight line (Fig. 4.14 page 32).

For this problem, the constant velocity spherical earth means that the two ray-paths will be to chords that subtend the two epicentral angles ($\Delta=60^\circ$ and 90°). The subtended angles are reckoned with respect to the center of the earth. Thus the two-chords that are the two raypaths for the two epicentral distances form two triangles: an equilateral triangle (all interior angles are 60°); and, a $90^\circ, 45^\circ, 45^\circ$ triangle. Furthermore, we know the sides of these two rectangles is just the radius of the earth (6400 km).

For the right triangle (epicentral distance $\Delta=90^\circ$), the Pythagorean theorem states that the hypotenuse squared is EQUAL to the square of the two sides added together.

$$a^2 + b^2 = c^2 \triangleright c = \sqrt{a^2 + b^2}$$

Thus the LENGTH of the hypotenuse (raypath) for the 90° epicentral distance right triangle is:

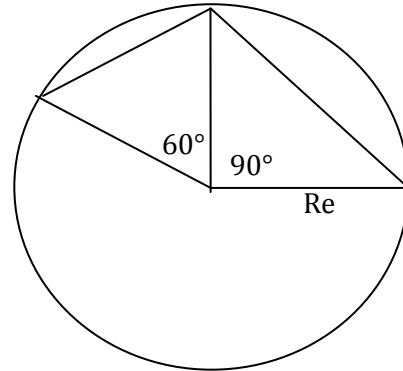
$$L_{90^\circ} = \sqrt{r_e^2 + r_e^2} = \sqrt{2r_e^2} = \sqrt{2} * r_e$$

And, the LENGTH of the chord (raypath) for the equilateral triangle is easy because all three sides of the triangle are the same length, which will be just the earth's radius r_e .

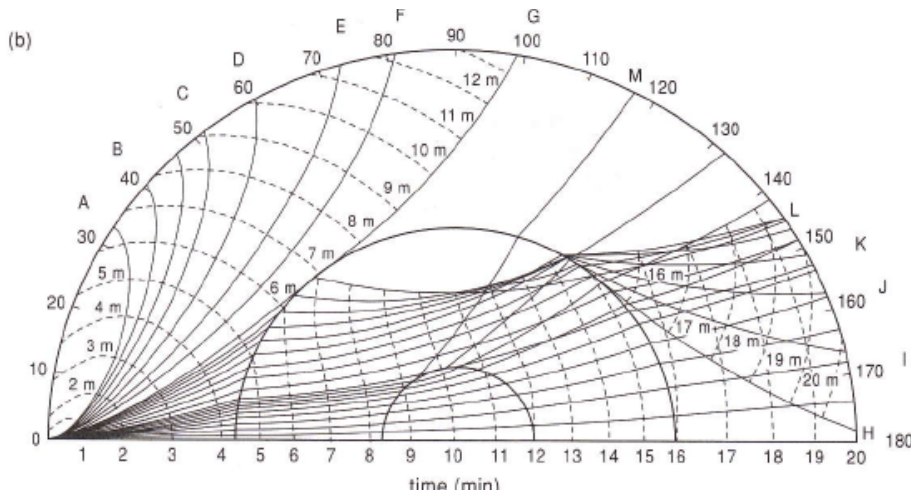
$$L_{60^\circ} = r_e.$$

Finally, remember that the travel time for a uniform velocity earth is just the length of the ray (which we just calculated) divided by the velocity. And, we were asked to calculate just the ratio of the travel-times along the two rays. So, calculate the ratio of the travel-times (T):

$$\frac{T_{90^\circ}}{T_{60^\circ}} = \frac{L_{90^\circ} / V}{L_{60^\circ} / V} = \frac{\sqrt{2} * r_e}{r_e} = \sqrt{2} = 1.414 \quad \therefore$$



13. Using figure 4.20, repeat the calculation of the travel-time ratio for real P-waves that travel through the real earth.



$$t(\Delta = 60^\circ) \approx 10 \text{ min.} \quad t(\Delta = 90^\circ) \approx 13 \text{ min.}$$

$$\frac{t(90^\circ)}{t(60^\circ)} = \frac{13}{10} = 1.3$$

The travel-time ratio of 1.3 is lower than the 1.41 travel-time ratio found for a constant velocity earth. Why is this? It is because the earth velocity structure is NOT constant but increase with depth, hence the $\Delta=90^\circ$ wave that dives deeper into the earth goes faster!

- 14. A ray, traveling down through the interior of a spherically layered planet, encounters a layer that extends from 3100 to 3000 km radius. If the velocities above, within, and below the layer are respectively 10,11,12 km/s, and the ray was incident to the layer top at 40°, then it leaves the bottom of the 11 km/s layer at what angle ?**

This problem is for waves propagating in a sphere, and not in a Cartesian space; therefore, the spherical ray tracing equations should be used (Eqn. 4.8).

$$\frac{r_1 \sin(\theta_1)}{v_1} = \frac{r_2 \sin(\theta_2)}{v_2}.$$

Define the given incident angle at the top of spherical shell: $r_1 = 3100$ km, $\theta_1 = 40^\circ$. Note that that the spherical equation simplifies to the Cartesian form if $r_1 = r_2$.

So, let solve for θ_2 which is the angle just under the top spherical shell:

$$\frac{3000 * \sin(\theta_1)}{10} = \frac{3000 * \sin(\theta_2)}{11} \triangleright \theta_2 = \sin^{-1}\left(\frac{11}{10} \sin(40^\circ)\right) = 45^\circ$$

Now, use the spherical Snell's law equation to calculate θ_3

$$\theta_3 = \sin^{-1}\left(\frac{r_1}{r_2} * \frac{v_3}{v_2} \sin(\theta_2)\right) = \sin^{-1}\left(\frac{3100}{3000} * \frac{12}{11} \sin(45^\circ)\right) = 53^\circ$$

The ray leaves the bottom of the 11 km/s layer at 53° with respect to the surface normal.

- 15. A rock has its rigidity (shear) modulus (μ) equal to $\frac{3}{4}$ of its bulk modulus (κ). If melting the rock does NOT change the bulk modulus or density (but does change the shear modulus), the ratio of V_p in the solid to the V_p in the partial molten state will be?**

The important point here is that the shear modulus for the melted rock will be assumed to be zero. As you recollect, an ideal fluid (melted rock) does not support any shear stress hence its shear modulus is zero.

Stated: $\mu = (3/4) \kappa$ and $\mu = 0$ for melted rock.

P-wave velocity equation:
$$V_p = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$

Now, form a ratio of the velocity of the solid rock to its liquid. We will square the ratio so we don't need to 'tote' around the square-root, and then take square-root at end of problem.

$$\frac{V_{p,solid}^2}{V_{p,liquid}^2} = \frac{\kappa + \frac{4}{3}\mu}{\rho} \bigg/ \frac{\kappa + 0*\mu}{\rho} = \frac{\kappa + \frac{4}{3}\mu}{\kappa} = \frac{2\kappa}{\kappa} = 2$$

$$\frac{V_{p,solid}^2}{V_{p,liquid}^2} = 2 \triangleright \frac{V_{p,solid}}{V_{p,liquid}} = \sqrt{2} = 1.41$$

END OF BOOK QUESTION; Extra Questions below.

1. What are the differences between a longitudinal and transverse wave in terms of their direction of propagation and vibration direction.
2. Draw a graph of the earth's P and S velocity structure.
3. If the velocity of a wave is constant, and its frequency is increased, write an expression that shows how the wavelength changes. Will the wavelength be bigger or smaller?
4. Draw a 1 hertz (1 cycle per second) sinusoidal wave with time along the x-axis and label the amplitude and period of the wave.
5. Draw a 1 meter long sinusoidal wave with distance along the x-axis and label the amplitude and wavelength of the wave.
6. When a P-wave moves through a medium at its P-wave velocity, what is the wave moving ?
7. What is the difference between a raypath and a wavefront. What is constant on the wavefront?
8. How does a seismometer work?